Visco-Elastic Fluid Flow Over a Non-Linearly Stretching Surface

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***Abstract* - we considered the visco-elastic flow over a non-linearly stretching surface. The effects of thermal radiation and viscous dissipation have been taken in consideration in the energy equation. The mathematical model has been solved numerically by shooting technique with higher order integration scheme.**

***Index Terms* - on-linearly stretching surface visco-elasticity.**

INTRODUCTION

In contrast to the well-known Blasius flow problem [2005] which involves laminar viscous boundary layer fluid flow above a fixed flat plate, the flow of a visco-elastic fluid over a rigid plate moving steadily in an otherwise quiescent fluid is sometimes referred to as Sakiadis flow [1961] after the pioneering work of that researcher.

Unfortunately, great deal of fluid flow applications in industrial processes are concerned with non-Newtonian and visco-elastic fluids, such as polymer melts and solution, heat-treated materials traveling between a feed roll or materials manufactured by extrusion, glass-fiber and paper production, cooling of metallic sheets or electronic chips, crystal growing and many others. In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching.

Crane [1970] first discussed the two-dimensional boundary layer flow caused by a linear stretching sheet in an otherwise quiescent fluid. He obtained a very simple closed form of exponential solution. The solution of the associated linear heat conduction equation was also presented by Crane [1970]. Afzal and Varshney [1980], Kuiken [1981] and Banks [1986] have considered the more general case of the sheet stretching with power-law velocity. Since fluid over stretching sheet has important industrial applications in polymer technology, where one deal with stretching of plastic sheet. During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The final product of desired characteristics strictly depends on the stretching rate, the rate of cooling in the process and the process of stretching. In view of these applications, consequently, the flow and heat transfer from a linearly stretching surface has attracted the attention of several researchers; Noor Afzal [1993] has obtained heat transfer from a stretching surface. The linear stretching problem has been extended for hydro-magnetic case by Chkrabarti and Gupta [1979], the boundary layer flow due to a plate stretching with power-law velocity distribution in the presence of a transverse magnetic field is studied by Chiam [1995]. Ali [1994] has reported flow and heat transfer characteristics on a stretched surface subjected to a power-law velocity and temperature distributions for three different boundary conditions. Furthermore, the flow field of a stretching wall with a Power-law velocity variation was discussed by Banks [1983]. Recently by Ali [1996], who extended Banks work for the stretched surface to be porous for different values of injections parameter.

The physical situation discussed in all the above studies is related to the process of linearly stretching sheet case. Another physical phenomenon is the case in which the sheet is stretched in a non-linear fashion. Gupta and Gupta [1977] have underlined that the stretching of the sheet may not necessarily be linear. In view of this, the nonlinearly stretching sheet was investigated by Vajravelu [1991]. Hence, it is interesting to study the flow and heat transfer phenomenon over a non- linearly stretching sheet. In the present chapter, we study the flow and heat transfer on a nonlinearly stretching sheet with velocity $u_w(x)$ for two different types of thermal boundary conditions on the sheets. Another effect which bear great importance on the heat transfer is the viscous dissipation which is also included in the energy equation.

FLOW ANALYSIS

We consider the flow of an incompressible visco-elastic fluid past a flat sheet coinciding with the plan *y=0*, the flow being confined to *y>0*.Two equal and opposite forces are applied along the *x*-axis so that the wall is stretched keeping the origin fixed. The steady two-dimensional boundary layer equations for this fluid in the usual notation are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\} \tag{2}$$

Where *(x, y)* denotes the Cartesian coordinates along the sheets and normal to it, *u* and *v* are the velocity components of the fluid in the *x* and *y* directions respectively and γ is the kinematic viscosity. The boundary conditions for the present problem are

$$u_{w(x)}=\frac{\upsilon}{L^{\frac{4}{3}}}x^{\frac{1}{3}},\quad v=0 \text{ at } y=0$$

$$u\to 0 \text{ as } y\to\infty \tag{3}$$

Where L is the reference length.

Defining new similarity variables

$$\eta=y\frac{x^{\frac{-1}{3}}}{L^{\frac{2}{3}}},\ u=\frac{\upsilon}{L^{\frac{4}{3}}}x^{\frac{1}{3}}f'(\eta),\ v=\frac{\upsilon}{L^{\frac{2}{3}}}x^{\frac{-1}{3}}\frac{(2f-\eta f')}{3} \tag{4}$$

With these changes of variables, equation (1) is identically satisfied and equation (2) is transformed into the following non-linear ordinary differential equation

$$f'^2-ff''=f'''-k_1\{2f'f'''-ff''''-f''^2\} \tag{5}$$

and the boundary conditions (3) becomes

$$f=0 \quad f'=1 \qquad at \quad \eta=0$$

$$f'\to 0 \quad f''\to 0 \quad as\ \eta\to\infty \tag{6}$$

The shear stress at the stretched surface is defined as

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_w \tag{7}$$

and we obtain from (4) and (7)

$$\tau_w=\mu\frac{\upsilon}{L^2}f''(0) \tag{8}$$

Where μ *is* the viscosity of the fluid.

Problem {(5)-using(6)} is solved numerically by employing a Runge-kutta algorithm for higher order initial value problems. Based on the numerical solution, we obtained, $f''(0)$ =-1.289747

HEAT TRANSFER ANALYSIS

By using usual boundary layer approximations, the equation of the energy for temperature *T* in the presence of radiation and viscous dissipation is given by

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial^2 y}+\mu\left(\frac{\partial u}{\partial y}\right)^2-\frac{\partial q_r}{\partial y} \tag{9}$$

Where ρ is the density, c_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid and q_r is the radiative heat flux. Using the Rosseland approximation for radiation (Siddheshwar. P.G et.al [2005]), the radiative heat flux is simplified as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{10}$$

Where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow such as that the term T^4 may be expressed as a linear function of temperature. Hence expanding T^4 in a Taylor series T_∞ *and* neglecting higher order terms we get

$$T^4\cong 4T_\infty^{\ 3}T-3T_\infty^{\ 4} \tag{11}$$

In view to equations (10) and (11), Equation (9) reduces to

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\left(k+\frac{16\sigma^*T_\infty^{\ 3}}{3\rho c_p k^*}\right)\frac{\partial^2 T}{\partial^2 y}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{12}$$

The thermal boundary conditions depends upon the type of the heating process being considered, here we considered two different types of heating process namely

(1) Prescribed surface temperature

(2) Prescribed power law heat flux

Case (1): Prescribed surface temperature (PST case)

In this circumstance, the boundary conditions are

$$T = T_w = T_\infty + A\left(\frac{x}{l}\right)^m \quad at \quad y = 0$$

$T \to T_\infty \quad as \quad y \to \infty$ (13)

T_w is wall temperature T_∞ is the fluid temperature far away from the surface, A is constant and m is wall temperature parameter. By considering $m=0$ and $A= T_w - T_\infty$ in equation (13) we obtain the constant temperature case. Defining the non-dimensional temperature

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{14}$$

Using equation (4), (13) and (14) in (12), we get

$$\theta'' + \frac{2k_0}{3}\Pr f\theta' - \Pr k_0 m f'\theta = -\Pr k_0 E_c (f'')^2 \tag{15}$$

Where $E_c = \dfrac{\upsilon^2 L^{m-\frac{8}{3}}}{Ac_p x^{m-\frac{2}{3}}}$ (16)

is the Eckert number, $\Pr = \dfrac{\upsilon}{\alpha}$ is the Prandtl number, $Nr = \dfrac{k^* k}{4\sigma^* T_\infty^{\,3}}$ is the radiation parameter and $k_0 = \dfrac{3Nr}{3Nr+4}$ is thermal radiation, the prime denotes differentiation with respect to η. Realizing that the x-coordinate cannot be eliminated from equation (15) when $m \neq \dfrac{2}{3}$. So, the temperature profile always depends on the x-coordinate.

Obviously, we get an x-independent similarity equation from the above when $m = \dfrac{2}{3}$ and this yields equation (15) as

$$\theta'' + \frac{2k_0}{3}\Pr(f\theta' - f'\theta) = -\Pr k_0 E_c^{*}(f'')^2 \tag{17}$$

Where $E_c^{*} = \dfrac{\upsilon^2}{Ac_p L^2}$

and that all solution are then of similar type.

The boundary conditions for $\theta(\eta)$ follow from (13) and (14) as

$\theta(\eta)=1 \quad at \quad \eta=0$

$\theta \to 0 \quad as \quad \eta \to \infty$ (18)

The rate of heat transfer of the surface is derived from equation (13) and (14) as

$$-k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -kA\theta'(0)\left(\frac{x}{L}\right)^{m-\frac{1}{3}} \cdot \frac{1}{L} \tag{19}$$

Where k is the thermal conductivity.

Case(2): Prescribed power law heat flux (PHF case)

In PHF case we define dimensionless new temperature variable as

$$g(\eta) = \frac{T - T_\infty}{\frac{D}{k} x^{m+\frac{1}{3}} L^{\frac{2}{3}-m}} \tag{20}$$

with the following boundary conditions

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_w = D\left(\frac{x}{L}\right)^m \quad at \quad y=0$$

$T \to T_\infty \quad as \quad y \to \infty$ (21)

Where D is constant and m=0 provides the constant heat flux case.

Using equation (4) and (20) in (12), we get

$$g'' + \frac{2k_0}{3}\Pr fg' - \Pr k_0 (m + \frac{1}{3}) f' g = -\Pr k_0 E_c (f'')^2$$

Where $E_c = \dfrac{\upsilon^2 L^{m-\frac{10}{3}}}{\frac{D}{k} c_p x^{m-\frac{1}{3}}}$ (22)

is the Eckert number, $\Pr = \dfrac{\upsilon}{\alpha}$ is the Prandtl number and $k_0 = \dfrac{3Nr}{3Nr+4}$ is thermal radiation, the prime denotes differentiation with respect to η.

Realizing that the x-coordinate cannot be eliminated from equation (22) when $m \neq \dfrac{1}{3}$. So, the temperature profile always depends on the x-coordinate.

Obviously, we get an x-independent similarity equation from the above when $m = \dfrac{1}{3}$ this yields equation (22) as

$$g'' + \frac{2k_0}{3}\Pr(fg' - f'g) = -\Pr k_0 E_c^*(f'')^2 \qquad (23)$$

Where $E_c^* = \dfrac{\upsilon^2}{Dc_p L^3}$

and that all solution are then of similar type.

The boundary conditions for $g(\eta)$ follow from (20) and (21) as

$g'(\eta)=-1$ at $\eta=0$

$g \to 0$ as $\eta \to \infty$ (24)

Equation (17) and (23) are the same and then we can summarize the complete heat transfer problem as

$$h'' + \frac{2k_0}{3}\Pr(fh' - f'h) = -\Pr k_0 E_c^*(f'')^2 \qquad (25)$$

$$E_c^* = \frac{\upsilon^2}{Ac_p L^2}$$

With h=*1 at* $\eta=0$; $h \to 0$ as $\eta \to \infty$ *and* for the PST case (m=2/3)

While $h'(\eta)= -1$ *at* $\eta=0$; $g \to 0$ *as* $\eta \to \infty$ *and* $E_c^* = \dfrac{\upsilon^2}{Dc_p L^3}$ for PHF case (m=1/3)

NUMERICAL SOLUTION

The procedure for completing the numerical solution for $h(\eta$,.there is no any analytical solution for the flow problem and, accordingly, one had to use numerical techniques. It is clear that $f''(0)$=-1.289747 in this problem by taking visco-elastic parameter $k_1=0.2$. Since the flow problem is uncoupled from the thermal problem, changes in the values of *Pr, Nr* and *Ec* will not affect the fluid velocity. For this reason, both the function *f* and its derivatives are identical in the complete problem (flow and heat transfer). In view of the above discussions, we have solved numerically, first the problem {(5)-(6)} which provide $f''(0)$ and second, with this result, we solve numerically heat transfer problem. This procedure has already been applied to discuss some flow and heat transfer problem (Cortell.R[2005b]). Equation (5) and (25) can easily be written as the first order system.

$$u_1' = u_2$$
$$u_2' = u_3$$
$$u_3' = u_4$$
$$u_4' = \frac{2k_1 u_2 u_4 - u_4 + u_2^2 - 2u_1 u_3}{2k_1 u_1}$$
$$u_5' = u_6$$
$$u_6' = -\frac{2}{3}\Pr k_0 (u_1 u_5 - u_2 u_4) - \Pr k_0 E_c^s u_3^2 \qquad (26)$$

Where the prime denotes differentiation with respect to η, $u_1 = f$, $u_5 = h$ *and the value of* $u_3(0) = f''(0)$ is given. Withal, in accordance with conditions (6) and (18) we obtain

u1(0)=0, u2(0)=1, u3(0)=-1.289747, u4(0)=$\frac{1}{1-2k1}$,u5(0)=1 (27)

u2(∞)=0,u3(∞)=0,u5(∞)=0 (28)

Using numerical methods of integration and disregarding temporarily the conditions (28), a family of solutions of {(26)-(27} can be obtained for arbitrarily chosen values of u_6 *(0)*. Tentatively we assume that a special values of $|\theta'(0)|$ yields a solution for which $\theta(\eta)$ *and* $\theta'(\eta)$ vanishes at a certain $\eta = \eta_\infty$ (condition (28)) and satisfies the additional condition

$$u_2(\eta_\infty) = 0, u_5(\eta_\infty) = 0 \qquad (29)$$

We guess u_5 (0) and integrate equation (26) with condition (27) as an initial value problem by employing Runge-kutta algorithm higher order initial value problems with the additional conditions (29). It is worth mentioning that, for each numerical solution, the η_∞ value depends on the non-dimensional parameters *Pr, Nr and Ec.*

RESULTS AND DISCUSSION

In this paper we investigated the visco-elastic boundary layer flow and heat transfer over a non-linear stretching sheet in the presence of radiation and viscous dissipation.

Fig-1 represents the influence of radiation parameter Nr on temperature distribution $\theta(\eta)$. From this figure, we observe that as the radiation parameter Nr increases, $\theta(\eta)$ decreases. This results qualitatively agrees with the fact of the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid.

Fig-2 represents the effect of Pr on temperature distribution $\theta(\eta)$. We infer from this figure that the temperature profile decreases with increase in Prandtl number (Pr). This is because of the fact that the thermal boundary layer thickness decreases with increase in Prandtl number (Pr).

Another effect, which bears great importance on heat transfer, is the viscous dissipation. When the viscosity of the fluid and/or the velocity gradient is high, the dissipation term becomes more important. The graph for temperature distribution $\theta(\eta)$ for different values of Eckert numbers is plotted in Fig-3, The effect of increasing *Ec* is to increase the magnitude of $\theta(\eta)$ in the flow region in both the cases of PST and PHF. This is due to the fact that heat energy is stored in the fluid due to frictional heating.

The graphs for the situation when the boundary has been prescribed with heat flux (PHF) are shown in fig.4-6. It is noticed from these figures that the wall temperature is not unity; it is changed at the wall with the change of physical parameters. We observe that thermal radiation parameter *Nr* and Prandtl number Pr and Eckert number *Ec* have same qualitative effects which we found in PST case but quantitatively wall temperature is more in PHF case.

SUMMARY AND CONCLUSIONS

We considered the visco-elastic flow over a non-linearly stretching surface. The effects of thermal radiation and viscous dissipation have been taken in consideration in the energy equation. The mathematical model has been solved numerically by shooting technique with higher order integration scheme.

The important findings of our study are as follows.

1. The effect of radiation parameter *Nr* is to decrease the temperature profile (both in PST and PHF case).
2. The effect of Eckert number *Ec* is to increase the temperature profile (both in PST and PHF case).
3. The effect of Prandtl number *Pr* is to decrease the temperature profile (both in PST and PHF case).

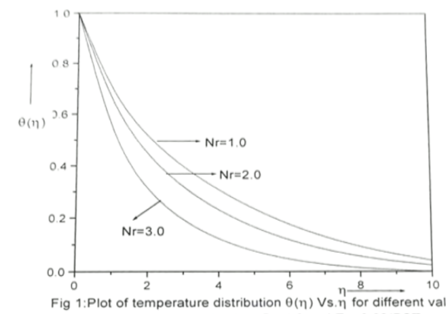

Fig 1:Plot of temperature distribution θ(η) Vs.η for different values of Radiation parameter Nr,when Pr=1.0 and Ec=0.02(PST case)

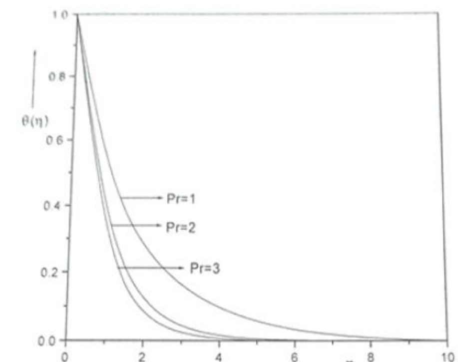

Fig 2: Plot of temperature distribution θ(η) Vs.η for different values of Pr,when Nr=1.0 and E_c=.02(PSTcase)

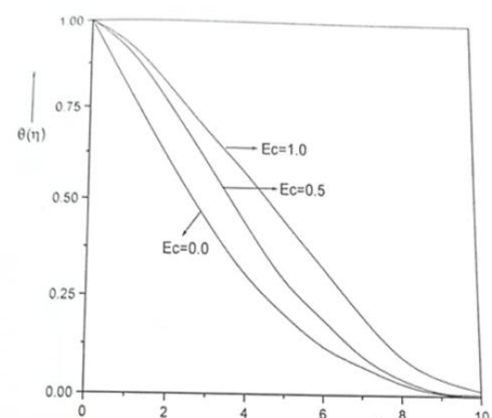

Fig 3:Plot of temperature distribution θ(η) Vs.η for different values Eckert number Ec,when Nr=Pr=1.0 (PSTcase)

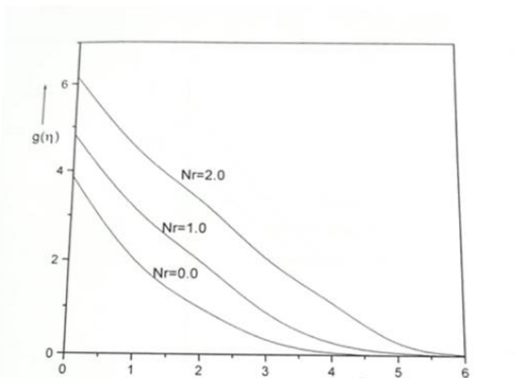

Fig 4:Plot of temperature distribution g(η) Vs.η for different values Radiation parameterNr,when Pr=2.0 and Ec=0.25(PHFcase)

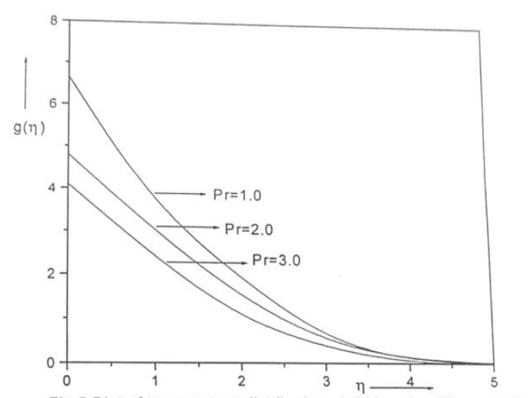

Fig 5:Plot of temperature distribution g(η) Vs.η for different value of Pr,when Nr=1.0 and Ec=0.25(PHF case)

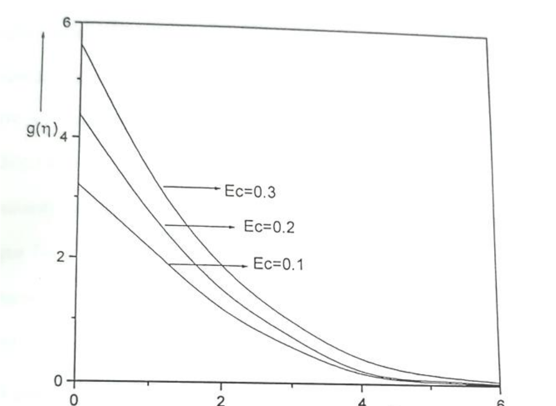

Fig 6: Plot of temperature distribution g(η) Vs.η for different values of Eckert numberEc,when Nr=Pr=1.0(PHFcase)

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